

Solutions to Maths workbook – 1 | Trigonometric Identities and Equations

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Daily Tutorial Sheet - 1

$$*1.(BC) \frac{1}{\cos \beta} = \frac{2 + \cos \alpha}{1 + 2 \cos \alpha} \Rightarrow \frac{1 - \cos \beta}{1 + \cos \beta} = \frac{1}{3} \left[\frac{1 - \cos \alpha}{1 + \cos \alpha} \right] \Rightarrow \tan^2 \left(\frac{\beta}{2} \right) = \frac{1}{3} \tan^2 \left(\frac{\alpha}{2} \right) \Rightarrow \sqrt{3} \tan \left(\frac{\beta}{2} \right) = \pm \tan \left(\frac{\alpha}{2} \right)$$

$$*2.(AB) f(\cos 4\theta) = \frac{2 \cos^2 \theta}{2 \cos^2 \theta - 1} = \frac{1 + \cos 2\theta}{\cos 2\theta} = \frac{1}{\cos 2\theta} + 1$$

$$\Rightarrow f(\cos 4\theta) = \pm \sqrt{\frac{2}{1 + \cos 4\theta}} + 1; \quad \therefore f\left(\frac{1}{3}\right) = \pm \sqrt{\frac{3}{2}} + 1$$

$$*3.(CD) \operatorname{cosec} \left(\theta + \frac{(m-1)\pi}{4} \right) \operatorname{cosec} \left(\theta + \frac{m\pi}{4} \right) \Rightarrow \frac{\sqrt{2} \left[\sin \left(\left(\theta + \frac{m\pi}{4} \right) - \left(\theta + \frac{(m-1)\pi}{4} \right) \right) \right]}{\sin \left(\theta + \frac{(m-1)\pi}{4} \right) \sin \left(\theta + \frac{m\pi}{4} \right)}$$

$$\Rightarrow \sqrt{2} \left[\cot \left(\theta + \frac{(m-1)\pi}{4} \right) - \cot \left(\theta + \frac{m\pi}{4} \right) \right] \Rightarrow \sqrt{2} \left[\cot \theta - \cot \left(\theta + \frac{3\pi}{2} \right) \right] = 4\sqrt{2}$$

$$\Rightarrow \sqrt{2} [\cot \theta + \tan \theta] = 4\sqrt{2} \Rightarrow \tan \theta + \cot \theta = 4 \Rightarrow \tan^2 \theta - 4 \tan \theta + 1 = 0 \Rightarrow \tan \theta = 2 \pm \sqrt{3}$$

As $\theta \in \left(0, \frac{\pi}{2} \right)$, So $\theta = \frac{\pi}{12}$ and $\frac{5\pi}{12}$

*4.(ACD)

Let us first analyse the interval for θ .

1. $\theta \in [0, 2\pi]$ 2. $\tan(2\pi - \theta) > 0$ and 3. $-1 < \sin \theta < \frac{-\sqrt{3}}{2}$

From these three conditions $\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3} \right) \dots (A)$

Now, solving given equation: $2 \cos \theta - 2 \cos \theta \sin \phi = 2 \sin \theta \cos \phi - 1$

$$\Rightarrow 2 \sin(\theta + \phi) = 1 + 2 \cos \theta \Rightarrow \sin(\theta + \phi) = \frac{1}{2} + \cos \theta$$

From (A): $0 < \cos \theta < \frac{1}{2} \Rightarrow \sin(\theta + \phi) \in \left(\frac{1}{2}, 1 \right)$.

Now it is given that $\phi \in [0, 2\pi]$, using it and (A)

We have $\frac{3\pi}{2} < \theta + \phi < \frac{11\pi}{3}$

In this range $\sin(\theta + \phi) \in \left(\frac{1}{2}, 1 \right) \Rightarrow \frac{13\pi}{6} < \theta + \phi < \frac{17\pi}{6} \quad \therefore \phi \in \left(\frac{\pi}{2}, \frac{4\pi}{3} \right)$.

We can clearly see that no value of ϕ from options (A), (C) and (D) satisfy given interval but values of option (B) satisfy given interval.

5.(3) From the given 3 equations, we get $(xyz)\sin 3\theta = (y+z)\cos 3\theta$ (1)

$$= 2y\sin 3\theta + 2z\cos 3\theta \quad \text{.....(2)}$$

$$= (y+2z)\cos 3\theta + y\sin 3\theta \quad \text{.....(3)}$$

$$\text{From (2) \& (3), we get } y\sin 3\theta = y\cos 3\theta \Rightarrow \tan 3\theta = 1 \{y \neq 0\}$$

$$\text{From (1) \& (3) } \tan 3\theta = -\frac{y}{z} \Rightarrow y+z=0$$

From (1), we get $x=0$

So, the solution will be of the type (x_0, y_0, z_0)

$$\text{With } x_0=0, y_0=-z_0$$

$$\text{Only requirement is: } \tan 3\theta = 1 \Rightarrow 3\theta = n\pi + \frac{\pi}{4}$$

$$\theta = \frac{n\pi}{3} + \frac{\pi}{12} \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4} \{n=0, 1, 2\}$$

6.(8) $\frac{5}{4} \left[1 - \sin^2 2x \right] + \left(1 - 2\sin^2 x \cos^2 x \right) + \left(1 - 3\sin^2 x \cos^2 x \right) = 2$

$$\Rightarrow \frac{5}{4} - \frac{5}{4} \sin^2 2x + 2 - 5\sin^2 x \cos^2 x = 2 \Rightarrow \frac{5}{4} - \frac{5}{4} \sin^2 2x - \frac{5}{4} \sin^2 2x = 0$$

$$\Rightarrow \sin^2 2x = \frac{1}{2} \Rightarrow 2x = n\pi \pm \frac{\pi}{4} \Rightarrow x = \frac{n\pi}{2} \pm \frac{\pi}{8}$$

$$\therefore x \in [0, 2\pi] \Rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8} \text{ and } \frac{15\pi}{8}$$

7.(7) Let $\frac{\pi}{n} = \theta \Rightarrow \frac{1}{\sin \theta} - \frac{1}{\sin 3\theta} = \frac{1}{\sin 2\theta} \Rightarrow \frac{2 \sin \theta \cos \theta}{\sin \theta \sin 3\theta} = \frac{1}{\sin 2\theta} \Rightarrow 2 \sin 2\theta \cos 2\theta = \sin 3\theta \left[\sin \theta \neq 0 \right]$

$$\Rightarrow \sin 4\theta = \sin 3\theta \Rightarrow 2 \sin \frac{\theta}{2} \cos \frac{7\theta}{2} = 0 \Rightarrow \sin \frac{\theta}{2} = 0 \text{ or } \cos \frac{7\theta}{2} = 0$$

$$\Rightarrow \frac{\theta}{2} = k\pi \text{ or } \frac{7\theta}{2} = (2\ell+1)\frac{\pi}{2} \therefore \theta = 2k\pi \text{ or } \frac{(2\ell+1)\pi}{7} \Rightarrow \frac{\pi}{n} = 2k\pi \text{ or } \frac{(2\ell+1)\pi}{7}$$

$$\Rightarrow n = \frac{1}{2k}, \frac{7}{2\ell+1}$$

Clearly $n > 3$ and $n \in \mathbb{Z} \Rightarrow n = \frac{1}{2k}$ is not possible and $n = 7$ for $\ell = 0$

8.(3) $\tan \theta = \cot 5\theta \Rightarrow \tan \theta = \tan \left(\frac{\pi}{2} - 5\theta \right) \Rightarrow \theta = n\pi + \frac{\pi}{2} - 5\theta \Rightarrow \theta = \frac{n\pi}{6} + \frac{\pi}{12}$

$$\text{As } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \Rightarrow \theta = \pm \frac{\pi}{12}, \pm \frac{\pi}{4}, \pm \frac{5\pi}{12} \quad \text{... (i)}$$

$$\text{Also, } \sin 2\theta = \cos 4\theta \Rightarrow \cos 4\theta = \cos \left(\frac{\pi}{2} - 2\theta \right) \Rightarrow 4\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta \right)$$

$$\Rightarrow \theta = (4n+1)\frac{\pi}{12} \text{ or } \theta = (4n-1)\frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{-\pi}{4} \quad \text{... (ii)}$$

There are 3 common values of θ from (i) and (ii)